Advances in Nuclear Physics

EDITED BY
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Erich Vogt

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ADVANCES IN NUCLEAR PHYSICS

VOLUME 3

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PREFACE

With the appearance of Volume 3 of our series the review articles themselves can speak for the nature of the series. Our initial aim of charting the field of nuclear physics with some regularity and completeness is, hopefully, beginning to be established. We are greatly indebted to the willing cooperation of many authors which has kept the series on schedule. By means of the "stream" technique on which our series is based – in which articles emerge from a flow of future articles at the convenience of the authors – the articles appear in this volume without any special coordination of topics. The topics range from the interaction of pions with nuclei to direct reactions in deformed nuclei.

There is a great number of additional topics which the series hopes to include. Some of these are indicated by our list of future articles. Some have so far not appeared on our list because the topics have been reviewed recently in other channels. Much of our series has originated from the suggestions of our colleagues. We continue to welcome such aid and we continue to need, particularly, more suggestions about experimentalists who might write articles on experimental topics.

M. Baranger
E. Vogt

June 13, 1969

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THE NUCLEAR THREE-BODY PROBLEM

A. N. Mitra

1. INTRODUCTION AND SUMMARY

When one speaks of the three-body problem, the first characteristic that comes to mind is its "insolubility." This describes the situation for the helium atom whose Schrödinger equation does not admit of an exact solution in the sense, say, of the corresponding hydrogen atom problem. The feature of insolubility thus is intimately associated with the very law of force—the Coulomb force—which so accurately describes the behavior of atomic systems. If this law were replaced by something simpler, say the harmonic oscillator force, insolubility would certainly not be a problem any more, though presumably more serious (physical) problems would arise. However, thanks to our better knowledge of atomic systems, this freedom simply does not exist. Therefore, the best the theoretical physicist can do with atomic three-body systems is to devise powerful approximation methods to obtain numerically accurate results for comparison with experiment. No one would seriously expect these methods, *by themselves*, to throw any new light over what is already known on the basic electromagnetic low of interaction which just happens to be too well established.

In the domain of nuclear interactions, one is not so fortunate. There is no well established law of interactions, and even questions of whether, or not, only two-body forces are basic, are not yet properly understood. Certain qualitative features are no doubt understood, such as the existence of a long-range attraction, a short-range repulsion, a tensor force of moderate range, and presumably a spin–orbit force of shorter range. These and other features of the two-body force have very significant effects, not only on the behavior of two-nucleon systems, but on more general systems consisting of more than two nucleons. More specifically, the *off-shell**

* A potential V, or the corresponding reaction matrix T, can be regarded as a matrix in energy space, so that its elements are of the form $\langle E \mid V \mid E' \rangle$; similarly $\langle E \mid T \mid E' \rangle$ are the corresponding elements of T. The "on-shell" elements are diagonal in this representation ($E = E'$) while the "off-shell" ones are nondiagonal ($E \neq E'$).

elements of a two-body potential have more readily observable effects on the behavior of $N(> 2)$ particles than the mere study of a two-particle system would reveal. The three-body system, being the simplest one with* $N > 2$, therefore affords a unique opportunity of learning more about the qualitative features of a two-body potential, and of deciding on questions like the adequacy of two-body forces for the description of more complex nuclear phenomena. This is where the nuclear three-body problem differs from the corresponding atomic problem. While powerful approximation methods are required for both systems, their *physical* scopes are wider for nuclear than for atomic systems in the context of our present-day knowledge of nuclear forces. For the three-nucleon system, these approximation methods are not merely tools for maintaining numerical accuracy, but instruments for providing reliable judgement on the *input* assumptions about the forces, as seen from a comparison of the predictions with observable quantities.

Traditionally, variational methods have served as the basic tools for three-body investigations. The earlier variational treatments,† however, suffered from a sort of slackness, presumably born out of indifference to the choice of the input potentials, but this nevertheless prevented a fair comparison of the *results* of the assumptions with experiment. This slackness has all along been manifest in the sharp contrast between the accurate variational functions used for atomic systems, and the corresponding functions of poorer quality chosen frequently for nuclear systems, inspite of the richer variety of two-body forces required for the latter. Essentially, an attitude of indifference had seemed to prevent variational investigations till rather recently, from extracting as much *physical* information about the efficacy of two-body forces from three-body studies as was warranted on physical grounds. For example, the structure of the Hamada–Johnston[1] potential which has been rather successful in correlating two-body phenomena, appears to be too elaborate for simple variational functions[2] to describe its full predictive powers which may well require more detailed

* Another system which is often claimed to be the simplest one for the study of off-shell effects is $(pp\gamma)$ which arises out of a p–p Bremsstrahlung process. Clearly, the two final protons, being part of a three-body system $(pp\gamma)$, contribute appreciable off-shell elements to the radiative cross section. The p–p system in the final state moreover gives a particularly "clean" realization of nuclear off-shell effects, since the electromagnetic part of the coupling is not only unambiguous but weak enough to be taken in first order.

† See any standard text book on nuclear physics for references and details on variational calculations upto the end of the last decade.

trial functions. However, the emergence of more accurate three-body techniques in the present decade, following on the new theory of Faddeév,[3] seems to have spurred variational investigators into greater activity, with the result that the gap between the "variational" and the so-called "exact" treatments has been narrowing down.* This trend might serve to emphasize the basic unity of purpose between variational and exact treatments of the three-body problem, namely, to learn more about the two-body interaction from a study of the properties of three-particle systems.

Regarding the "exact" methods in three-body investigations, it is clearly necessary to define their limitations, since the problem is, in fact, insoluble with convectional interactions. However, it is possible to imagine certain simplified forms for the two-body interactions which would facilitate a three-body treatment in an exact fashion. For example, the harmonic oscillator potential is known to be a soluble problem for *any number* of *bound* particles (not just three), and thus would be an ideal choice, had it not been for its incompatibility with a simultaneous treatment of scattering problems. δ-function potentials provide a second example of exact solution for the three-body problem, but for the difficulty of infinite binding energy. However, such a potential is perfectly compatible with the scattering of a "nucleon" by a "deuteron." Indeed, δ-function potentials were used by Skoroniakov and Ter-Martirosian[5] to present perhaps the first *exact* formulation of three-particle scattering in a satisfactory fashion. The only reason why it was forgotten was that the interaction was too idealized for a realistic treatment of the three-body problem. A similar fate befell another exact but highly idealized formulation of the three-body problem[6] using a "boundary-value potential." It must be emphasized, however, that these earlier formulations had laid considerable gound work for future investigations in the task of finding more acceptable forms of interactions, ones which would be simple enough for a three-body treatment, yet realistic enough for a detailed fit to two-body data for both bound and scattering states. The clue to such a possibility was really contained in the work of Yamaguchi[7] who had earlier discovered that the two-body problem happened to have an explicit *algebraic* solution with separable potentials which could be made realistic enough to fit two-body data in sufficient details through the inclusion of tensor forces.[8] It was, therefore, logical to explore the possibilities for a three-body treatment with such potentials, and it turned

* It is understood that the latest value of the triton binding energy obtained by Blatt and collaborators using the Hamada–Johnston potential is short of experiment by about 1.5 MeV.[4]

out that the three-body problem could indeed be reduced to the simplicity of an *effective-two-body problem* with ordinary potentials.[9] As a word of caution, one should note, however, that the use of such artificial-looking potentials can be defended only on the pedagogical ground that our ignorance of the *true* force between two nucleons might as well be turned to our advantage for the time being, as long as the observable effects on two-body systems are at least approximately reproduced.

The limitations of such an approach are fairly clear. We must not, for example, deceive ourselves that we are solving the three-body problem exactly. By truncating the two-body potential suitably, we are merely reducing the magnitude of the three-body problem, so that no further approximation on the way to its final solution may be necessary. This is *not* a proper substitute for the *true* potential and the only justification for resorting to it is that the latter is not really known. In a variational treatment, one chooses simple trial functions for studying the effects of conventional (local) potentials.[1] For an "exact" treatment one needs to truncate the potential in a suitable manner. We shall discuss more about the choice of the potential as we go along, but at this stage it should suffice to say that the basic motivation is to preserve as much of the observable two-body properties as possible without making the interaction too complicated. The emphasis on exactness in such an approximation is mainly for the sake of convenience in numerical computation (in as much as the two-body problem is easier to handle than the three-body one), and not merely on "exactness" for its own sake. In this respect this new approach is no more basic than the (more conventional) variational treatments.

Unfortunately, the philosophy of "exactness" has been carried rather too far in recent years. The spectacular advances in the mathematical theory of scattering operators, following the work of Faddeév,[3] Lovelace,[10] and Weinberg,[11] seem to be giving a totally different turn to the three-body problem in nuclear physics from its traditional role as a probe into the properties of the two-body potential.* The emphasis in three-body investigations has been shifting more and more toward the exploration of formal properties of scattering amplitudes, existence of various poles (real and fictitious), multiparticle unitarity on and off the energy shell, and so on.† Within such an elaborate framework, the essential physics behind the three-body problem has necessarily had to be confined to very simple (rather

* Equations similar to Faddeév–Lovelace had earlier been obtained by Watson (see, for example, Ref. 12) in connection with his multiple scattering theory.

† For a recent review of multiparticle scattering theories, and for a detailed list of references, see Watson and Nuttall.[13]

idealized) structures of the input potentials, so that these could lend themselves to easy mathematical manipulation. On the other hand, such simple choices could not possibly provide any new *physical* information over what would be expected with less sophisticated techniques (e.g., the Schrödinger equation), using the same input information. One must, in other words, be prepared to *put in* more information (through the interactions), in order to expect more by way of output. These input informations must take the form of additional effects such as the tensor force, short range repulsion, etc., parametrized in a suitable manner, and inserted into the three-body framework. It seems to the author that such effects are rather difficult to put into the rather elaborate framework of Faddeév–Lovelace[3,10] or Amado,[14] where the emphasis is more on the scattering amplitudes than on the wave function. On the other hand, the Schrödinger equation, which emphasizes the wave function, rather than the scattering amplitudes, seems to provide a much easier, and more natural, basis for the introduction of such effects through appropriate additions to the two-body potentials. This is understandable, for the Faddeév equations[3] were originally motivated by a desire to eliminate the potentials[3] in favor of the reaction matrices, in order that the scheme might prove a better starting point for the treatment of high energy phenomena where the potential concept is not particularly useful. On the other hand, the tensor force or the hard core are typically "potential" concepts, which have a more natural place in the language of the Schrödinger equation. To introduce such effects in the Faddeév language, one must, so to speak, work a few steps backward. The elaborate manipulations needed to define a wave function in the Faddeév language,[15] would serve to illustrate this point.

There exists a vast amount of literature on the "exact" theory* of the three-body problem, which has been the subject of extensive investigations in the present decade, both from the point of view of the Schrödinger equation[9,16,17] as well in terms of the Faddeév theory under the separable approximation.[10,14] It would be simply confusing to try to present all the details of the various methods which overlap considerably in spirit if not in letter. Broadly speaking, the separable approximations to the Faddeév theory have generally been confined to the simplest assumptions which may be paraphrased as equivalents to *rank-one*, *s*-wave, separable potentials. These methods, which have been extensively reviewed in the literature,† seem to give interesting results for n–d scattering[19] and break-up reactions.[20,21]

* This word is used mainly for notational convenience, without fear of confusion, since its limitations have already been described in the preceding paragraphs.

† See, for example, Ref. 18.

However, the approximations used in these methods do not seem to be adequate for the *bound* three-body problem, or even the zero energy *n–d* system, which show a more sensitive dependence on the details of the two-nucleon interaction, such as the tensor force and the repulsive core.

In this article we shall be concerned mainly with the development of the three-body problem from the point of view of potentials, rather than the reaction-matrices. The basic dynamics will be governed by the three-body Schrödinger equation, rather than the Faddeév equations. The latter will be shown to follow from the former through simple algebraic manipulations with separable potentials which will form the basic dynamical ingredients. The wave function will play a central role in this development and various quantities of physical interest, scattering amplitudes, bound-state parameters and so on, will be derived from simple quantum mechanical principles through the use of appropriate boundary conditions on the wave function. The principles of application will first be illustrated through some simple examples of model three-body systems before discussing the more realistic case of the three-nucleon problem. For the latter, the emphasis will be mainly on the bound state and the nature of the information that its properties are able to provide on the details of the two-nucleon interaction. No attempt will be made to describe anew the results of *n–d* scattering or break-up reactions[19–21] which have been extensively discussed in Ref. 18. The review will also include a brief discussion of physical systems which can be approximately regarded as three-body problems so as to be amenable to the techniques of this article.

The article does not make any claims to completeness, either as a detailed mathematical theory, or with regard to a listing of the references. The emphasis being on the applicability of the techniques, the plausibility of various steps are defended mainly on simple, pedagogical grounds. In keeping with this attitude, excessive algebraic details will be avoided through appropriate references to the literature. It is hoped that some of the techniques described in the following sections will also prove useful for applications beyond the topics actually covered, especially in related areas of nuclear physics where the physical conditions warrant the assumption of approximate three-body structures.

We conclude this section with a summary of contents of the article. Since separable potentials will form an integral part of the formalism, Section 2 gives a brief account of these potentials, their general structures in momentum space and the actual forms of parametrization for several situations of physical interest. Section 3 develops a model three-body problem of three identical particles interacting through *s*-wave potentials, so as first to illustrate the principles of handling the three-nucleon problem within the new formalism before getting involved into the actual details of more realistic systems. A detailed interpretation of the three-body wave function in terms of certain substructures brings out rather clearly the concept of resonating group structures first proposed by Wheeler.[22] In particular, the wave function of each particle with respect to the centre of mass of the other two, which we define as the "spectator function," is shown to bear the entire burden of dynamics in the process of reduction of the three-body problem to an equivalent two-body problem. This quantity is also shown to provide the main clue for demonstrating the equivalence of this formalism with the separable approximation to the Faddeév theory. For further illustration of the techniques of the present formalism, Section 4 deals with a model "stripping problem" for a "deuteron" consisting of two identical particles impinging on an infinitely heavy "nucleus," to bring out the types of physical effects which a "three-body treatment" of the stripping problem can hope to incorporate. The amplitudes for scattering and stripping processes are derived within this framework through suitable boundary conditions on the "spectator functions," in an "exact" fashion as well as under the DWBA approximation, and the results compared. Section 5 outlines the treatment of the more physical three-nucleon problem which takes account of the symmetries under the spin and isospin degrees of freedom. The structures of the spectator functions, as well as the integral equations they satisfy, are indicated for both doublet and quartet states. General properties of the bound-state wave function are discussed and the methods of evaluation of physical quantities such as the probabilities of various angular momentum states, charge and magnetic form factors of H^3 and He^3, are broadly indicated. Section 6 gives a broad assessment of the numerical results for the binding energy of H^3 and the doublet *n–d* scattering length, obtained under successive refinements of the potentials, and the main conclusions (together with limitations) that these results warrant. In particular, it is shown that *both* the tensor force and the repulsive core in the $^1S_0$ state are strongly indicated by the results. This section also includes a brief discussion of the results for the electromagnetic radii of He^3 and H^3, the Coulomb energy of He^3, and the relativistic corrections to the binding energy of H^3. Section 7 discusses the possibility of treating certain light nuclei and hypernuclei as approximate three-body systems and indicates the type of results already obtained by various authors through such studies. Finally, Section 8 summarizes the limitations of this method and contemporary ones with regard to their domains of applicability.